Homework #2 (10 points) - Show all work on the following problems:

Problem 1 (3 points): Find the mutual inductance of two circular loops of wire, both horizontally oriented (in the x-y plane). A big loop with radius b has its center at the origin. A small loop with radius a is located above it, at height z above the origin.

1a (1 point). If current I flows through the big loop, find the magnetic flux through the small loop (assume $a \ll z$, so that the magnetic field from the big loop at its location is essentially constant and equal to the field on the z-axis).

1b (1 point). If current I flows through the small loop, find the magnetic flux through the big loop (assume $a \ll z$, so that you can approximate its field as a dipole).

1c (1 point). Calculate the mutual inductances M_{12} and M_{21}.

Problem 2 (2 points): Calculate the self-inductance per unit length for a coaxial cable. As a model for the cable, assume that current flows uniformly through a circular cross section off radius R, and then returns along the surface of the cable (so that there is no magnetic field outside the cable). You can neglect the thin insulator that separates the current from the return current in your calculations.

Problem 3 (3 points): Imagine a perfect conductor, with σ infinite.

3a (1 point): Show that the magnetic field is constant inside a perfect conductor.

3b (1 point): Show that the magnetic flux through a perfectly conducting loop is constant.

3c (1 point): In a "superconductor", the magnetic flux is not only constant, but is identically zero. Show that any current in a superconductor can flow only on its surface.

Problem 4 (2 points): Let us discuss an infinite wire running along the z-axis. This wire has a current I that varies with z (but not t), and a charge density λ that varies with t (but not z). Assume that $I(z = 0) = \lambda(t = 0) = 0$.

4a (1 point): Show that the only consistent solution has $\lambda(t) = kt$ and $I(t) = kz$, with k a constant, by looking at the charge flowing in a segment dz during a time dt.

4b (1 point): In the quasi-static approximation, the fields would be $E = \frac{\lambda}{2\pi\varepsilon_0 s}\hat{s}$, $B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$, with λ and I from the above. Show explicitly that these quasi-static solutions satisfy all four of Maxwell's equations (with all terms!), and thus are in fact the exact solutions.